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A COMPARISON OF BINARY P.C.M. AND F.M. FOR SOUND SIGNAL DISTRIBUTION

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J.O. Drewery, M.A.Cantab.

D. Maurice
Head of Research Department

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SUMMARY

The relationships between output signal-to-noise ratio, link signal-to-noise ratio and link bandwidth are derived for distribution of audio signals by either f.m. or binary p.c.m. systems. The calculation includes the effect of aural sensitivity weighting of the noise spectrum, and in the case of p.c.m. a fixed digit error rate is assumed. Comparison of the two systems is made on the basis of the signal power required for a given link bandwidth and noise spectral density. The information is presented graphically so that the choice of system requiring less signal power can be made quickly.

1. INTRODUCTION

Frequency modulation and pulse code modulation (p.c.m.) are both coding systems which require a greater bandwidth for transmission than the 'linear' modulation systems a.m. d.s.b. and its derivatives, but which are able to produce a better signal-to-noise ratio at the output of a demodulator under a given link signal-to-noise condition than linear systems. In effect, they exchange link bandwidth for signal-to-noise ratio, which linear systems are unable to do. The question arises as to whether or not for a given link bandwidth, link noise and required output signal-to-noise ratio the performance of a f.m. system is superior to that of a p.c.m. system. The following simple treatment attempts to answer this question for the case of a single channel for which no cross-talk problems are involved. The effect of f.m. pre- and de-emphasis is neglected since it is generally agreed that the overall gain in signal-to-noise ratio obtainable by this means is only 2 to 3 dB. Also, in the case of p.c.m. the use of various artifices for improving output signal-to-noise has not been considered.

2. SIGNAL-TO-NOISE IMPROVEMENT IN F.M.

In the treatment of modulation systems it is customary to regard the message path between modulator and demodulator as a link or channel possessing a definite bandwidth and spectral distribution of noise. If the noise is assumed white (and gaussian) then the total link mean noise power $N_l = N_d B$ where N_d is the mean noise spectral density (i.e. the noise power per unit bandwidth) and B is the channel bandwidth. In the case of f.m. the signal power is equal to the unmodulated carrier power since modulation merely redistributes the spectral energy of the carrier. Thus the link signal-to-noise ratio is equal to the carrier-to-noise ratio.

The ability of f.m. to exchange bandwidth for link signal-to-noise ratio is demonstrated by the so-called signal-to-noise improvement equation[1] which relates the output signal-to-noise ratio of a f.m. demodulator to the input link carrier-to-noise ratio for a sinusoidal modulation waveform. This equation is

$$\left(S/N_o\right) = \frac{3}{2}\left(\Delta F/f_m\right)^2\left(B/f_m\right)\left(C/N_l\right) \qquad (1)$$

where ΔF is the peak deviation, f_m is the maximum modulation frequency, C is the mean carrier power, S is the mean output signal power and N_o is the mean output noise power. The derivation of equation (1) assumes that the demodulator contains a low-pass filter of cut-off frequency f_m in its output and that intermodulation products between the noise components can be neglected compared with noise modulation of the carrier so that only the link noise power within a bandwidth $2f_m$ contributes to the output noise.

Confusion sometimes arises because the signal-to-noise improvement equation may be defined in terms of the ratio of carrier to equivalent a.m. i.f. noise, N_{eq}.

$$\left(S/N_o\right) = 3\left(\Delta F/f_m\right)^2\left(C/N_{eq}\right) \qquad (2)$$

The equivalent a.m. i.f. noise is that link noise contained in the bandwidth $2f_m$ about the carrier frequency and would be the only noise to affect an a.m. transmission on the same link. This second definition of signal-to-noise improvement is thus useful in comparing the signal-to-noise improvements of f.m. and a.m. over the same link. The term carrier-to-noise ratio as generally understood, however, refers to the total link noise and consequently equation (1) will be used in the following treatment.

If the approximation is made that the modulation index, β, is much greater than 1 for all baseband frequencies then

$$B = 2\ \Delta F \qquad (3)$$

and equation (1) becomes

$$\left(S/_{N_o}\right) = \frac{3}{8}\left(B/_{f_m}\right)^3\left(C/_{N_l}\right) \qquad (4)$$

Hence it is clear that a decrease in (C/N_l) can be compensated by an increase in B. The assumption β>>1 holds in practice at 100% modulation, for all audio baseband frequencies except the highest octave. The output signal-to-noise ratio and the carrier-to-link noise ratio may be expressed in decibels as

$$\left(S/_{N_o}\right)_{dB} \text{ and } \left(C/_{N_l}\right)_{dB} \text{ respectively}$$

so that from equation (4)

$$\left(S/_{N_o}\right)_{dB} = -4{\cdot}3 + 30\ \log_{10}\left(B/_{f_m}\right) + \left(C/_{N_l}\right)_{dB} \qquad (5)$$

It is customary in the BBC to specify audio signal-to-noise ratios in terms of the quasi-peak reading obtained on a p.p.m., the noise being weighted according to a standard aural sensitivity curve. If the link noise is white and Gaussian, the demodulator output noise is Gaussian in amplitude and has a parabolic power spectrum. The effect of the aural sensitivity weighting on a parabolic power spectrum terminating sharply at 15 kHz has been found to add a correction of only 0·06 dB which can be ignored. It has been established empirically that a p.p.m. indicates a signal-to-noise ratio which is 5 dB less than that indicated by a r.m.s. meter irrespective of the noise spectrum provided its time amplitude is Gaussian. When corrected for aural sensitivity and p.p.m. reading, therefore, equation (5) becomes:

$$\left(S/_{N_o}\right)_{dB} = 30\ \log_{10}\left(B/_{f_m}\right) + \left(C/_{N_l}\right)_{dB} - 9{\cdot}3 \qquad (6)$$

3. SIGNAL-TO-NOISE IN BINARY P.C.M.

In the case of p.c.m. the output signal-to-noise ratio obtained from the decoder is independent of the link noise provided that the signal power from the encoder is adequate. The effect of the link noise is to introduce sporadic errors in the output signal rather than a continuous noise. The frequency of these errors, which are catastrophic, falls very quickly when the signal power rises above a threshold level and can be made very low by only a slight increase in the signal power. However, the process of signal quantization allows the transmission of signal samples with a limited accuracy and introduces a quantizing noise into the recovered output signal. This noise, which is a result of the error between the quantized signal values and the true signal values, can be reduced by finer spacing of the quantizing levels but this requires more digits per sample and the coded signal thereby occupies a larger bandwidth. It can easily be shown[2] that the relation between output signal to quantizing noise and number of digits, n, for a binary p.c.m. system is given by

$$\frac{\text{Peak-to-peak signal voltage}}{\text{Peak-to-peak q-noise voltage}} = 2^n \qquad (7)$$

Expressed in decibels

$$\left(\frac{\text{Peak-to-peak signal voltage}}{\text{Peak-to-peak q-noise voltage}}\right)_{dB} = 20\ \log 2^n = 6n \quad (8)$$

When allowance is made for the r.m.s. values of signal and noise, aural sensitivity weighting and p.p.m. correction the signal-to-noise ratio in dB as measured on a p.p.m. is given by

$$\left(S/_{N_o}\right)_{dB} = 6n - 7{\cdot}4 \qquad (9)$$

Since the signal must be sampled at a minimum rate of $2f_m$ samples/sec. the p.c.m. signal contains at least $2nf_m$ digits/sec. This signal requires a minimum transmission bandwidth of nf_m Hz when the digit pulses have a $\sin x/x$ shape and are spaced so that the peak of any one pulse occurs at the time of zero crossings of all the rest. The pulses may then be detected by sampling at the peak instants provided that no distortion has occurred.

Because of the susceptibility of $\sin x/x$ pulses to distortion, it is common practice to transmit $\sin^2 x$ pulses which are less difficult to detect. If the pulses are spaced so that successive pulse peaks occur at the instants of the previous pulse zeros then the bandwidth required is $2\ nf_m$ Hz. Thus

$$B = 2nf_m \qquad (10)$$

and using equation (9)

$$\left(S/_{N_o}\right)_{dB} = 3\left(B/_{f_m}\right) - 7{\cdot}4 \qquad (11)$$

As previously stated the signal power required for transmission is independent of the required output signal-to-noise ratio but must exceed a certain threshold value set by the link noise. P.C.M. signal-to-noise ratios are usually quoted in terms of peak power

to r.m.s. noise. In comparing the performance of p.c.m. and f.m. systems, however, it is more logical to work with the mean signal power which, for a unipolar binary p.c.m. system is equal to half the peak power. For an error rate of 1 in 10^{12}, which is sufficiently low for most practical purposes, the mean power to noise ratio must be 20 dB. Thus if P is the mean p.c.m. power

$$10 \log_{10} \left({}^{P}/_{N_l} \right) = \left({}^{P}/_{N_l} \right)_{\mathrm{dB}} = 20 \qquad (12)$$

4. COMPARISON OF SYSTEMS

Equations (6), (11) and (12) enable a comparison to be made. The relevant parameters are seen to be

$$\left({}^{S}/_{N_o} \right)_{\mathrm{dB}}, \left({}^{B}/_{f_m} \right), \left({}^{C}/_{N_l} \right)_{\mathrm{dB}} \text{ and } \left({}^{P}/_{N_l} \right)_{\mathrm{dB}}$$

Of these, (B/f_m) and C could be regarded as independent variables. However the link noise N_l is a function of link bandwidth B. Thus

$$N_l = N_d B = N_d f_m \left({}^{B}/_{f_m} \right) \qquad (13)$$

The link noise spectral density N_d may have any value greater than the absolute thermal value kT. To obtain a clearer picture, therefore, normalized link variables can be defined as:

$$b = {}^{B}/_{f_m} \qquad (14)$$

$$\text{and } n_d = 10 \log_{10} (N_d/kT) \qquad (15)$$

b is always greater than unity and n_d is always greater than zero. Normalized signal powers may also be defined with reference to the baseband thermal noise power

$$c = 10 \log_{10} C/kTf_m \qquad (16)$$

$$\text{and } p = 10 \log_{10} P/kTf_m \qquad (17)$$

so that $$\left({}^{C}/_{N_l} \right)_{\mathrm{dB}} = 10 \log_{10} \left({}^{C}/_{N_l} \right)$$

$$= 10 \log_{10} \frac{C}{kTf_m} - 10 \log_{10} \frac{N_o f_m b}{kTf_m}$$

$$= c - n_d - 10 \log_{10} b \qquad (18)$$

and $\left({}^{P}/_{N_l} \right)_{\mathrm{dB}}$ becomes

$$10 \log_{10} \left({}^{P}/_{N_l} \right) = p - n_d - 10 \log_{10} b \qquad (19)$$

Using these variables, equations (6), (11) and (12) become

$$\left({}^{S}/_{N_o} \right)_{\mathrm{dB}} = 30 \log_{10} b + c - n_d - 10 \log_{10} b - 9{\cdot}3$$

$$= 20 \log_{10} b + c - n_d - 9{\cdot}3 \qquad \text{for f.m.} \qquad (20)$$

and

$$\left.\begin{aligned} \left({}^{S}/_{N_o} \right)_{\mathrm{dB}} &= 3b - 7{\cdot}4 \qquad (21) \\ p &= 20 + n_d + 10 \log_{10} b \qquad (22) \end{aligned}\right\} \text{ for p.c.m.}$$

One way of interpreting these equations is to regard b and n_d as independent variables for both p.c.m. and f.m., $\left({}^{S}/_{N_o} \right)_{\mathrm{dB}}$ as a dependent variable for p.c.m. and f.m., p as a dependent variable for p.c.m. and c as a further independent variable for f.m. Two three-dimensional spaces may thus be drawn up, one representing $\left({}^{S}/_{N_o} \right)_{\mathrm{dB}}$ as a function of b and n_d, with a series of surfaces for various values of c for f.m., and a single plane surface for p.c.m., the other representing p as a function of b and n_d. If this interpretation is pursued it will be seen that for a given pair of values of b and n_d a required value of $\left({}^{S}/_{N_o} \right)_{\mathrm{dB}}$ can always be attained using f.m. at a certain value of c; however, the required value of $\left({}^{S}/_{N_o} \right)_{\mathrm{dB}}$ may be above the p.c.m. plane and thus be unattainable with p.c.m. If the value of $\left({}^{S}/_{N_o} \right)_{\mathrm{dB}}$ lies below the p.c.m. plane the required f.m. carrier power, c, must be compared with the mean p.c.m. power, p, deduced from the second three-dimensional space.

If a signal with a specified output signal-to-noise ratio is to be transmitted down a link having parameters b and n_d and it is required to discover which system is superior then two cases arise.

(a) The output signal-to-noise ratio cannot be achieved with p.c.m. because it exceeds the value given by equation (21). The output signal-to-noise ratio can be achieved with f.m. at a certain carrier power given by equation (20). Therefore f.m. must be used.

(b) The output signal-to-noise ratio can be exceeded using p.c.m. and achieved with f.m. at a certain

carrier power. In this case the system requiring less power should be chosen.

These cases may be presented graphically as follows:

(a) Required $\left(S/N_o\right)_{dB} > 3b - 7{\cdot}4$

In this case f.m. must be used and the necessary carrier power is given by

$$c = \left(S/N_o\right)_{dB} - 20 \log_{10} b + n_d + 9{\cdot}3 \qquad (23)$$

This could be plotted with $\left(S/N_o\right)_{dB}$ and n_d as parameters but requires several graphs.

A second important quantity which is, however, independent of the parameter n_d is the carrier-to-noise ratio on the link. This ratio must exceed the threshold value of 12 dB required by a f.m. demodulator. From equation (18)

$$\left(C/N_l\right)_{dB} = c - n_d - 10 \log_{10} b$$
$$= \left(S/N_o\right)_{dB} - 30 \log_{10} b + 9{\cdot}3 \qquad (24)$$

Fig. 1 shows the relationship between carrier-to-noise ratio and link bandwidth for various values of $\left(S/N_o\right)_{dB}$. The dotted line represents the boundary between cases (a) and (b) so that the graph is only relevant to case (a) in the area to the left of the line. The figure shows that for very small link bandwidths and low output signal-to-noise ratios the required carrier-to-noise ratio may well be below 12 dB. In such cases the carrier power must, of course, be increased.

(b) Required $\left(S/N_o\right)_{dB} < 3b - 7{\cdot}4$

Here p.c.m. and f.m. both possible

$$c = \left(S/N_o\right)_{dB} - 20 \log_{10} b + n_d + 9{\cdot}3$$

$$p = 10 \log_{10} b + n_d + 20$$

from equations (23) and (22) respectively.

The excess of f.m. power over p.c.m. power in dB is therefore

$$c - p = \left(S/N_o\right)_{dB} - 30 \log_{10} b - 10{\cdot}7 \qquad (25)$$

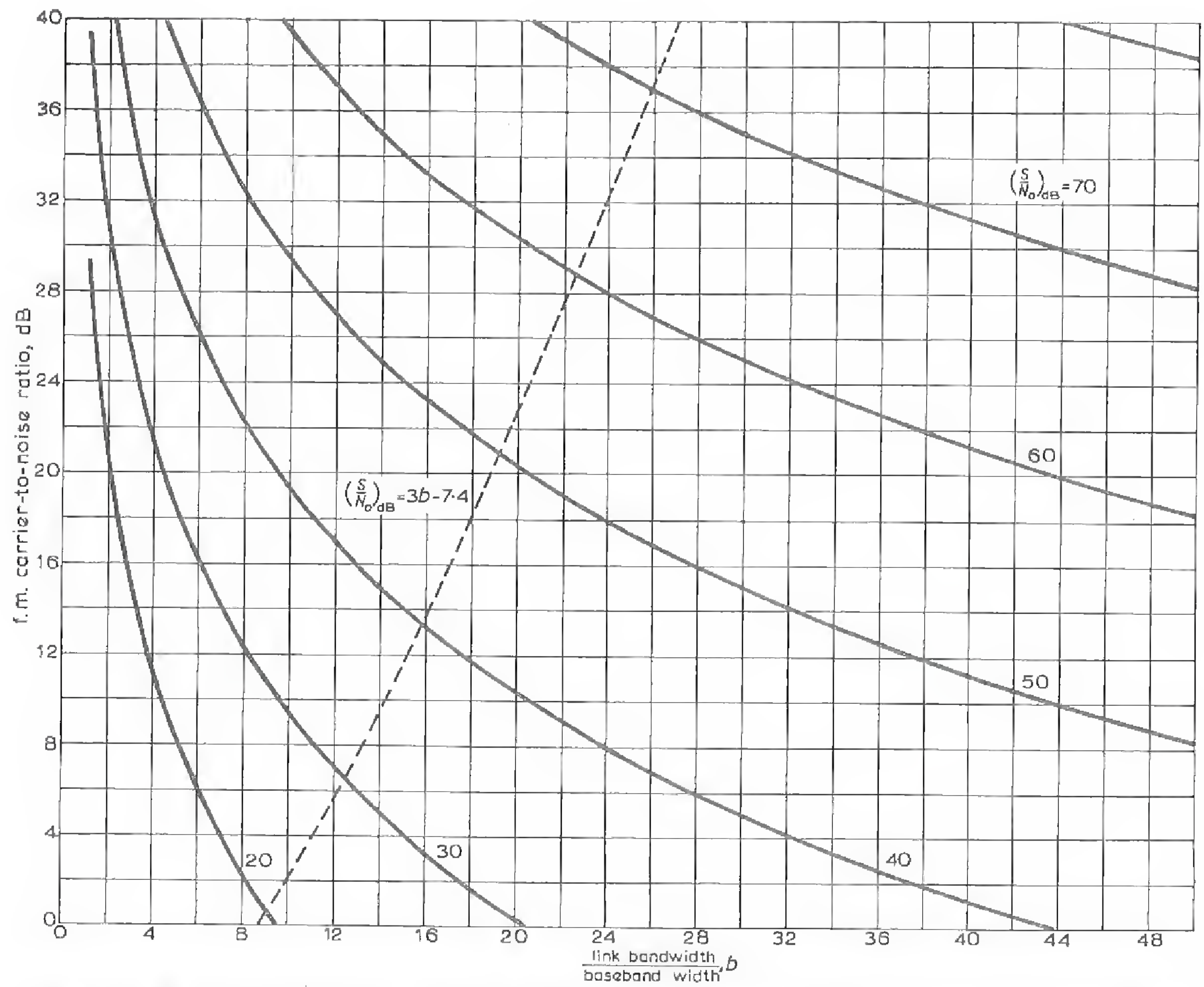

Fig. 1 - F.M. carrier-to-noise ratio as a function of link parameters and output signal-to-noise ratio

Fig. 2 shows this relationship which determines the system to be used. The dotted line represents the boundary between cases (a) and (b). If p.c.m. is used the required mean power is given by equation (22) which is plotted in Fig. 3 with link noise spectral density as parameter. If f.m. is used the required carrier power is given by equation (23) or, indirectly, by equations (25) and (22) and can thus be read off from the graphs of Figs. 2 and 3.

The f.m. carrier-to-noise is given, as before, by equation (24) and is plotted in the right hand region of the graph of Fig. 1.

As an example, suppose a link is available with a bandwidth of 20 times the baseband and a receiver of noise factor 10 dB in an otherwise perfect system. The link parameters are thus $b = 20$ and $n_d = 10$ dB. If the required $\left(S/N_o\right)_{\mathrm{dB}}$ is 60 then $\left(S/N_o\right)_{\mathrm{dB}} > 3b - 7{\cdot}4$ and case (a) applies. F.M. must be used and from equation (23) the required carrier power is 53 dB relative to baseband-thermal noise power. From Fig. 1 the carrier-to-noise ratio on the link is 30 dB which is well above the f.m. threshold value of 12 dB.

On the other hand if a greater bandwidth is available so that $b = 24$, then $\left(S/N_o\right)_{\mathrm{dB}} < 3b - 7{\cdot}4$ and case (b) applies. From Fig. 2 the f.m. power required is 8 dB greater than the p.c.m. power and therefore p.c.m. should be used. From Fig. 3 the p.c.m. power is 44 dB relative to baseband-thermal noise power and since the mean-power-to-noise ratio is 20 dB, the link noise power relative to baseband-thermal noise power is 24 dB. This is composed of 10 dB due to the imperfection of the link and 14 dB due to the bandwidth expansion factor.

5. CONCLUSIONS

Graphs have been provided which enable a choice of f.m. or p.c.m. systems to be made knowing the link parameters and required output signal-to-noise ratio. As a general observation, f.m. is superior to p.c.m. if high output signal-to-noise ratios are required but only limited bandwidth is available.

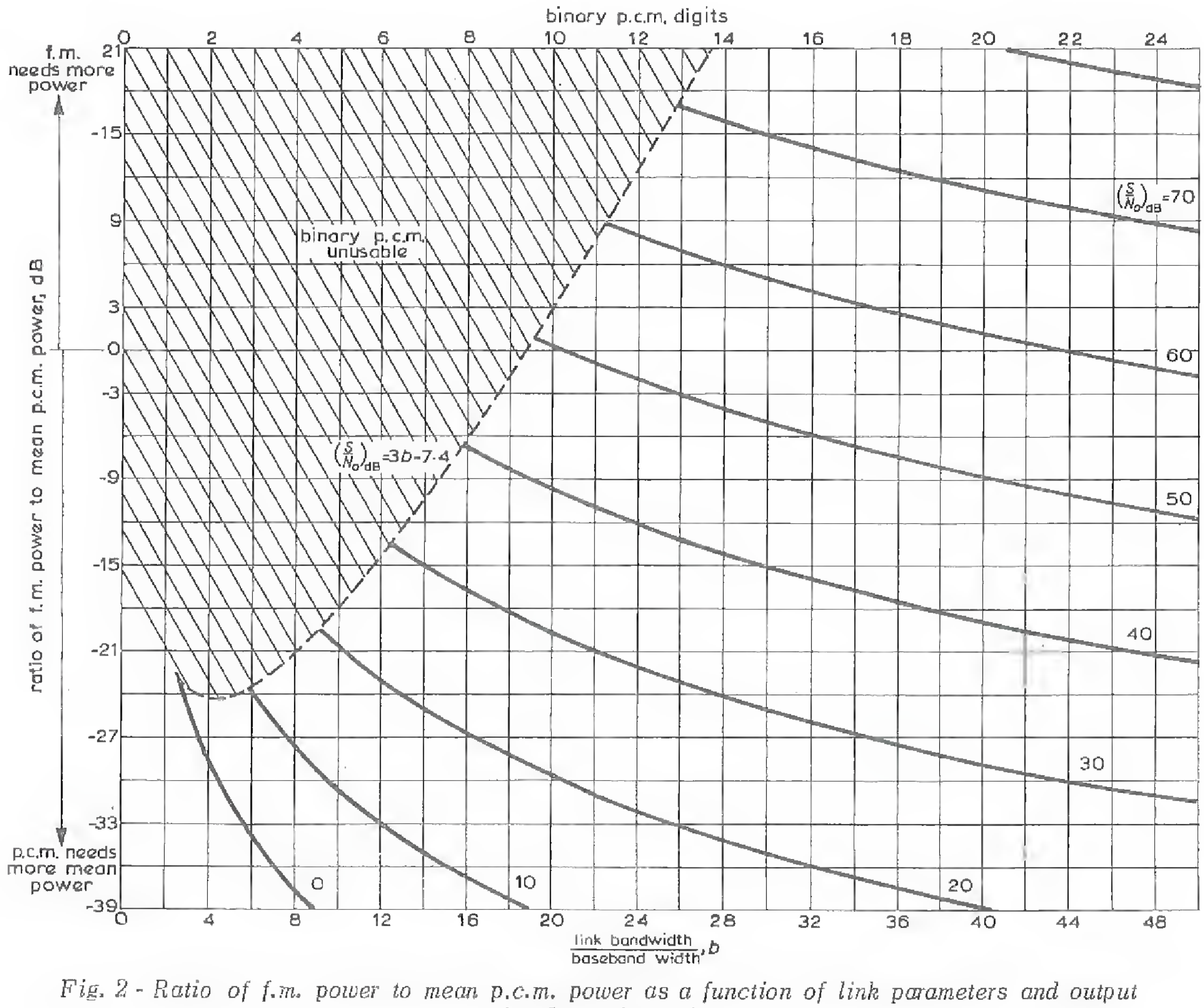

Fig. 2 - Ratio of f.m. power to mean p.c.m. power as a function of link parameters and output signal-to-noise ratio

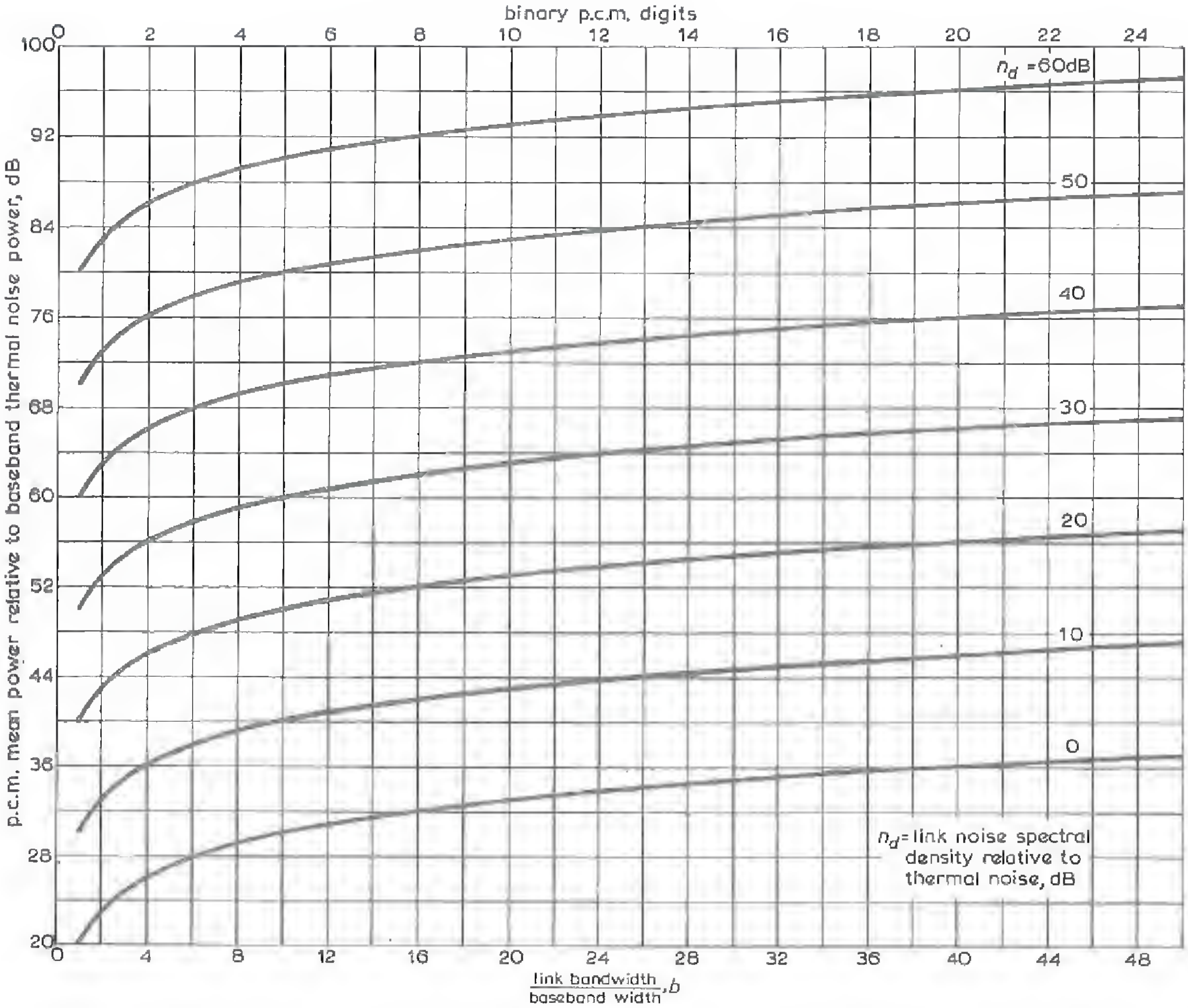

Fig. 3 - P.C.M. mean power as a function of link parameters

The derivation of the graphs has assumed an audio baseband because of the noise weighting network used in Sections 2 and 3. However, if any other baseband is assumed and the output noise weighted accordingly, this will result only in the addition of constants to equations (6) and (11) and the general form of the results will remain unchanged.

6. REFERENCES

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2. PANTER, P.F. 1965. Modulation, noise and spectral analysis. New York, McGraw-Hill, 1965, Chapter 20.

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